

Math 213, Fall 2025, Exam 1
Read Carefully, Write Neatly, Sketch Accurately,
Only simplify as needed (e.g. you may leave messy square roots)

1. (42 points) For the points $P(2,1,1)$, $Q(4,2,3)$ and $R(6,4,1)$
 - a. (4 points) Find the parametric equations of the line segment connecting P and Q .
 - b. (4 points) Find the distance between the points P and Q .
 - c. (4 points) Find the unit vector in the direction $\overrightarrow{PQ}$
 - d. (8 points) Find the area of the triangle with vertices P , Q , and R .
 - e. (8 points) Find the projection of $\overrightarrow{PQ}$ onto $\overrightarrow{PR}$
 - f. (4 points) Find the equation of the plane in which the 3 points P , Q , and R lie.
 - g. (10 points) Find the distance from the point $(6,3,2)$ to this plane.
2. (8 points) For the equation $x^2 + 2y^2 - 4x - 12y - z^2 + 22 = 0$, identify and sketch the surface. Explain cross sections perpendicular to any axes that might help clarify understanding your sketch.
3. (32 points) Let C be a curve parametrized by some $\mathbf{r}(t)$ where its velocity vector is given by $\mathbf{v}(t) = \mathbf{i} + t\sqrt{2}\mathbf{j} + t^2\mathbf{k}$ and the curve passes through the point $\mathbf{r}(3) = \mathbf{i} + 4\sqrt{2}\mathbf{j} + 5\mathbf{k}$
 - a. (6 points) Find the curve's path, $\mathbf{r}(t)$.
 - b. (4 points) Find the parametric equations for the line tangent to C at the point where $t = 3$.
 - c. (8 points) Find the length of the curve for $t \in [0,3]$.
 - d. (14 points) Find the curvature κ at $t = 3$
4. (10 points) Find an equation for, and sketch the graph of, the level curve of the function $f(x, y) = \sqrt{y + x^2} - 3$ that passes through the point $(-1, 3)$.
5. (8 points) Use vector algebra to show that a parallelogram with diagonals of equal lengths is a rectangle. (Hint: Let the vectors representing the sides be $\mathbf{u}$ and $\mathbf{v}$. Express the diagonals in terms of $\mathbf{u}$ and $\mathbf{v}$ and derive the consequence of the diagonals having equal lengths).